



*The Abdus Salam  
International Centre for Theoretical Physics*



**SMR/1845-16**

**Conference on Structure and Dynamics in Soft Matter and  
Biomolecules: From Single Molecules to Ensembles**

*4 - 8 June 2007*

**Structure and dynamics in the physics of foams**

Denis WEAIRE  
*School of Physics  
Trinity College  
Dublin 2  
IRELAND*

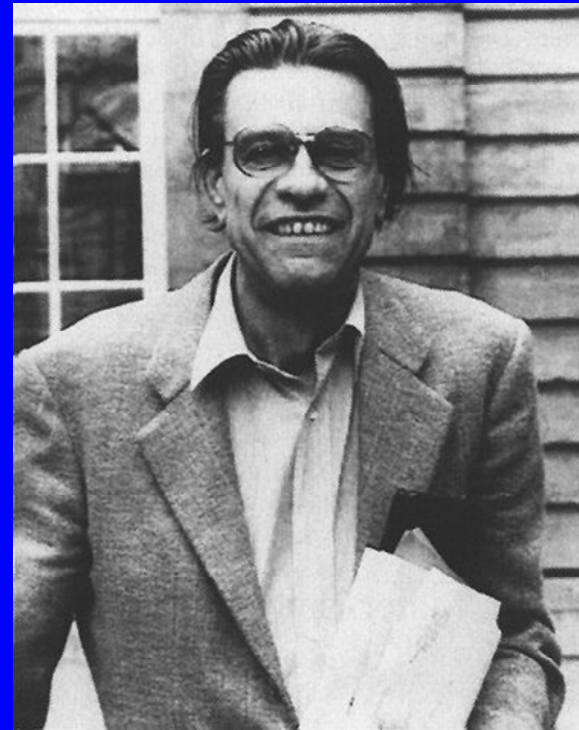
In memoriam

# Pierre Gilles de Gennes

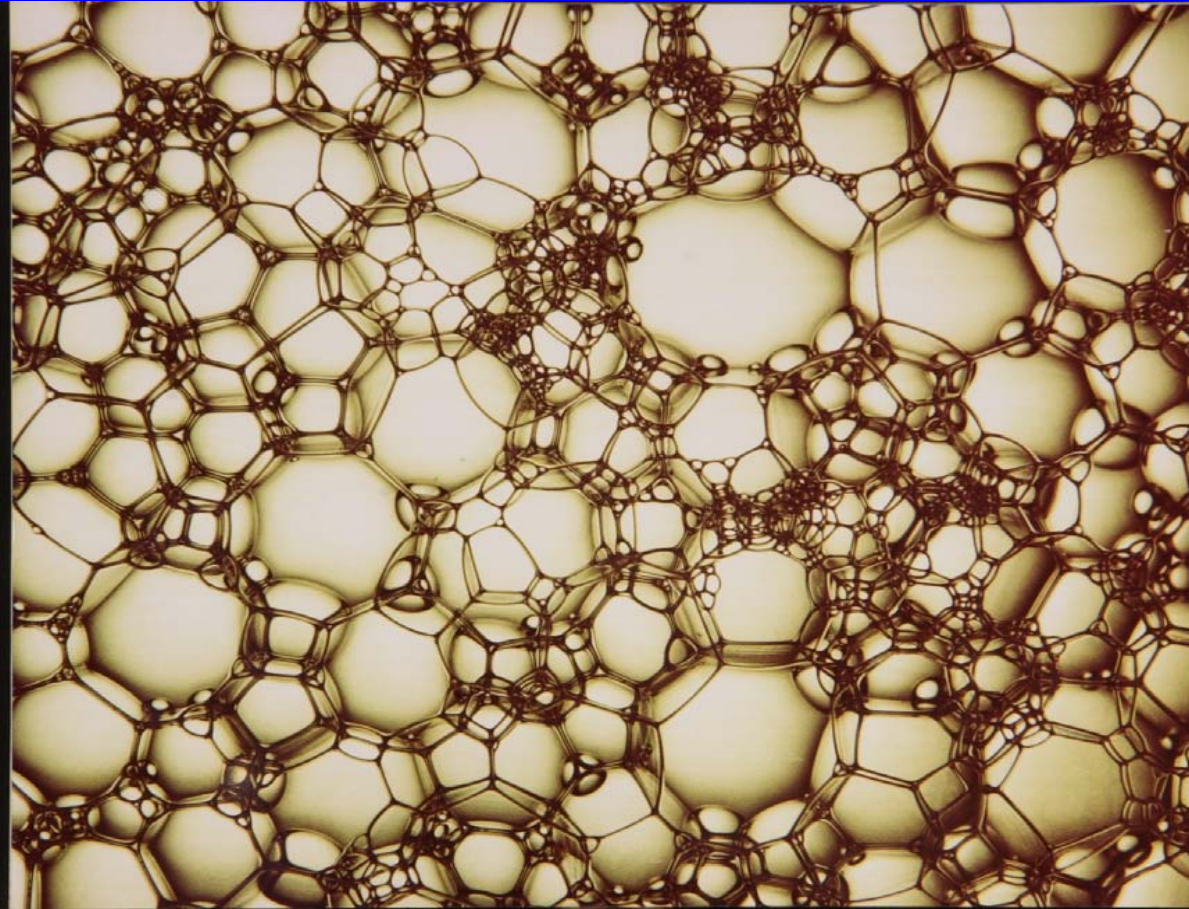
Nobel Prize Lecture

*Amusons-nous. Sur la terre et sur  
l'onde  
Malheureux qui fait son nom!  
Richesse, Honneurs, faux éclat de ce  
monde,  
Tout n'est que boules de savon.*

[Let's have fun. On land and sea,  
Fame brings nought but troubles,  
Riches, honours, vain celebrity,  
Are only soapy bubbles.]



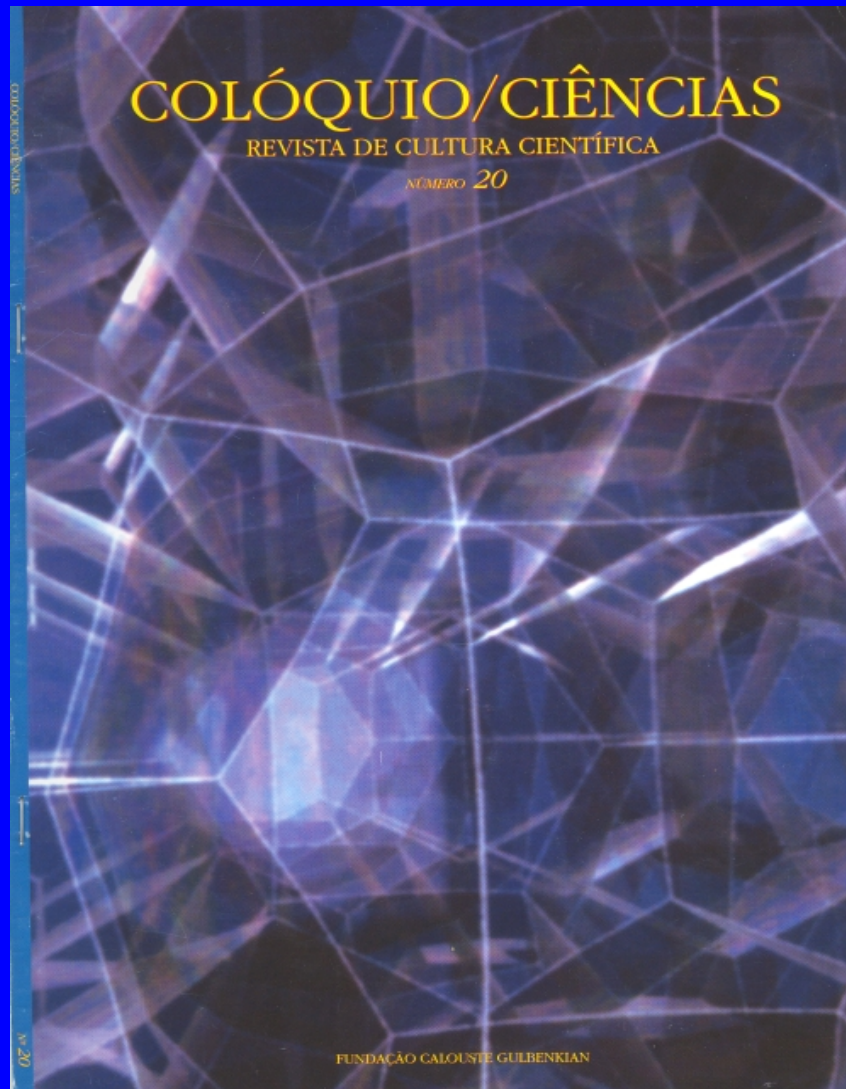
# Foams (in particular, rheology)



# Success story: statics/quasistatics

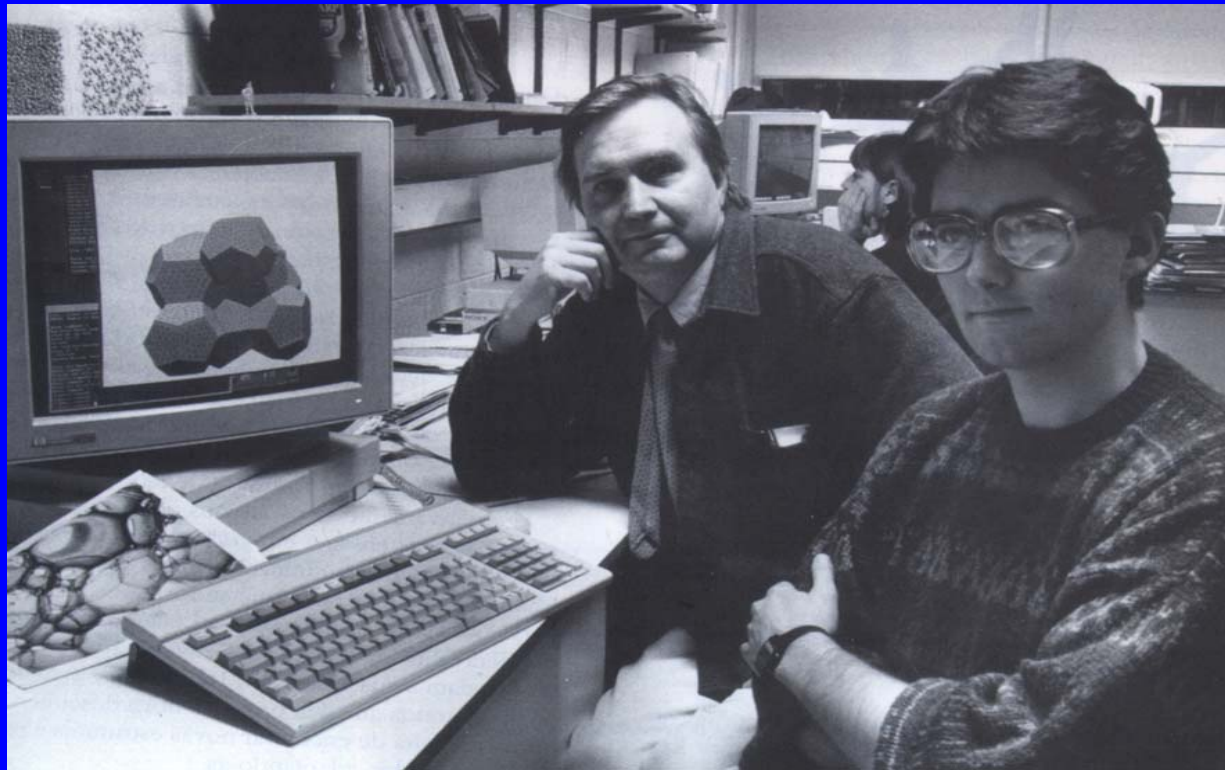
We understand many aspects of :

- structure
- elastic behaviour, onset of plasticity
- coarsening
- drainage
- electrical/thermal conductivity
- *etc*



# WEAIRE-PHELAN STRUCTURE

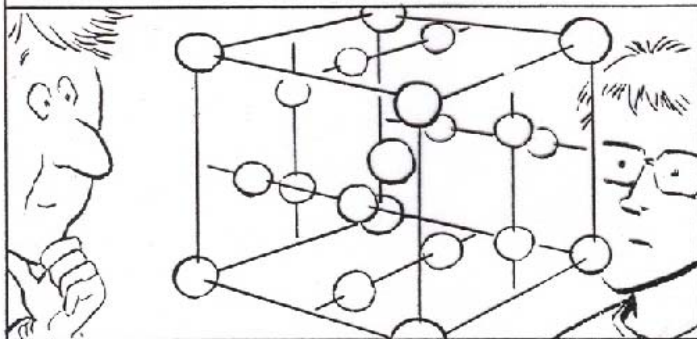




BUT SINCE KELVIN, HUGE STRIDES HAVE BEEN MADE IN CRYSTALLOGRAPHY. AFTER ALL, HE WAS WORKING BEFORE X-RAYS, BEFORE COMPUTERS...



IN LATE 1993 IRISH PHYSICISTS DENIS WEAIRE AND ROBERT PHELAN BUILT A NEW FOAM WHOSE SKELETON WAS A CRYSTAL STRUCTURE CALLED BETA-TUNGSTEN.

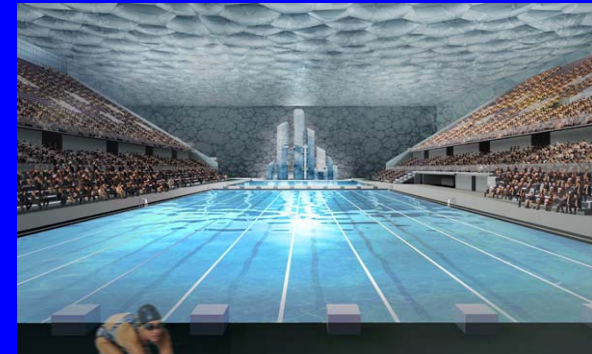
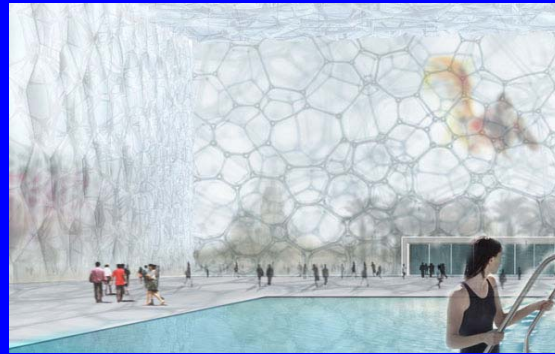


DRAWING A CELL AROUND EACH ATOM, WEAIRE AND PHELAN THEN USED "SURFACE EVOLVER" SOFTWARE TO EQUALIZE CELL VOLUMES AND FIT THEM TOGETHER PERFECTLY.

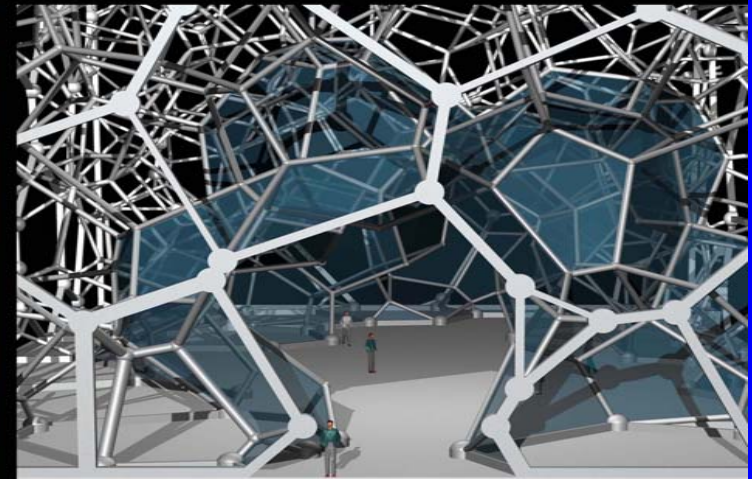
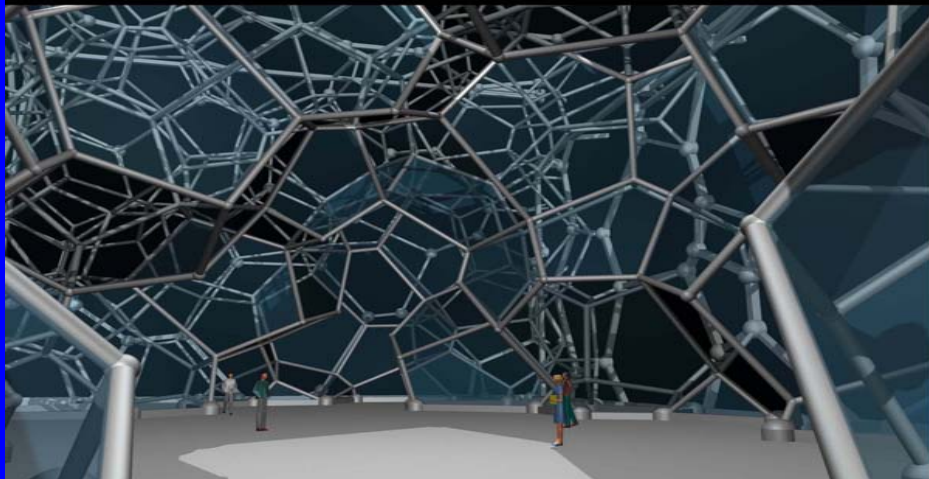
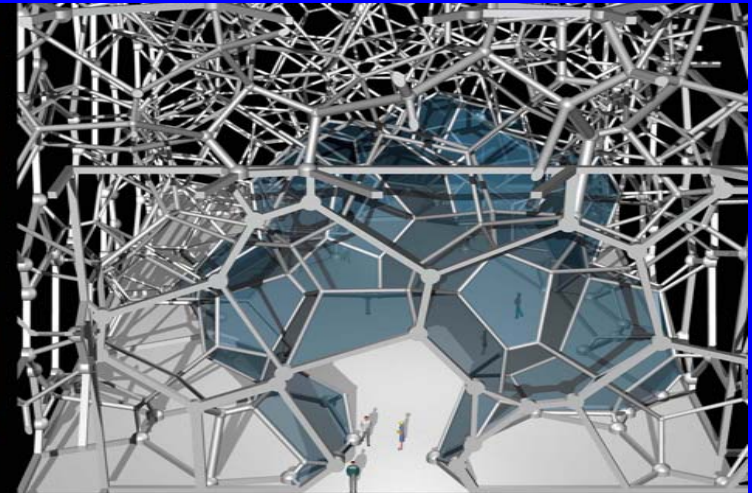
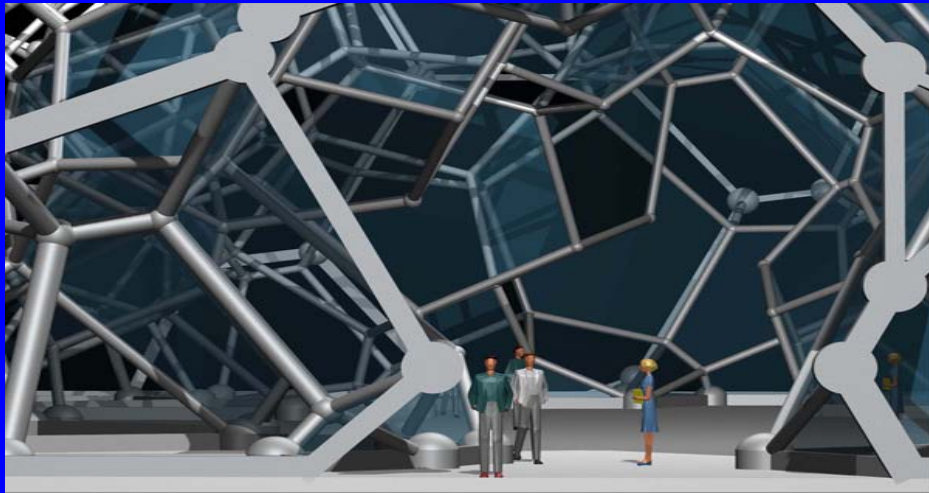


# The Water Cube

Arup's winning design for the 2008 Beijing Olympics,  
National Swimming Centre







Beijing National Swimming Centre

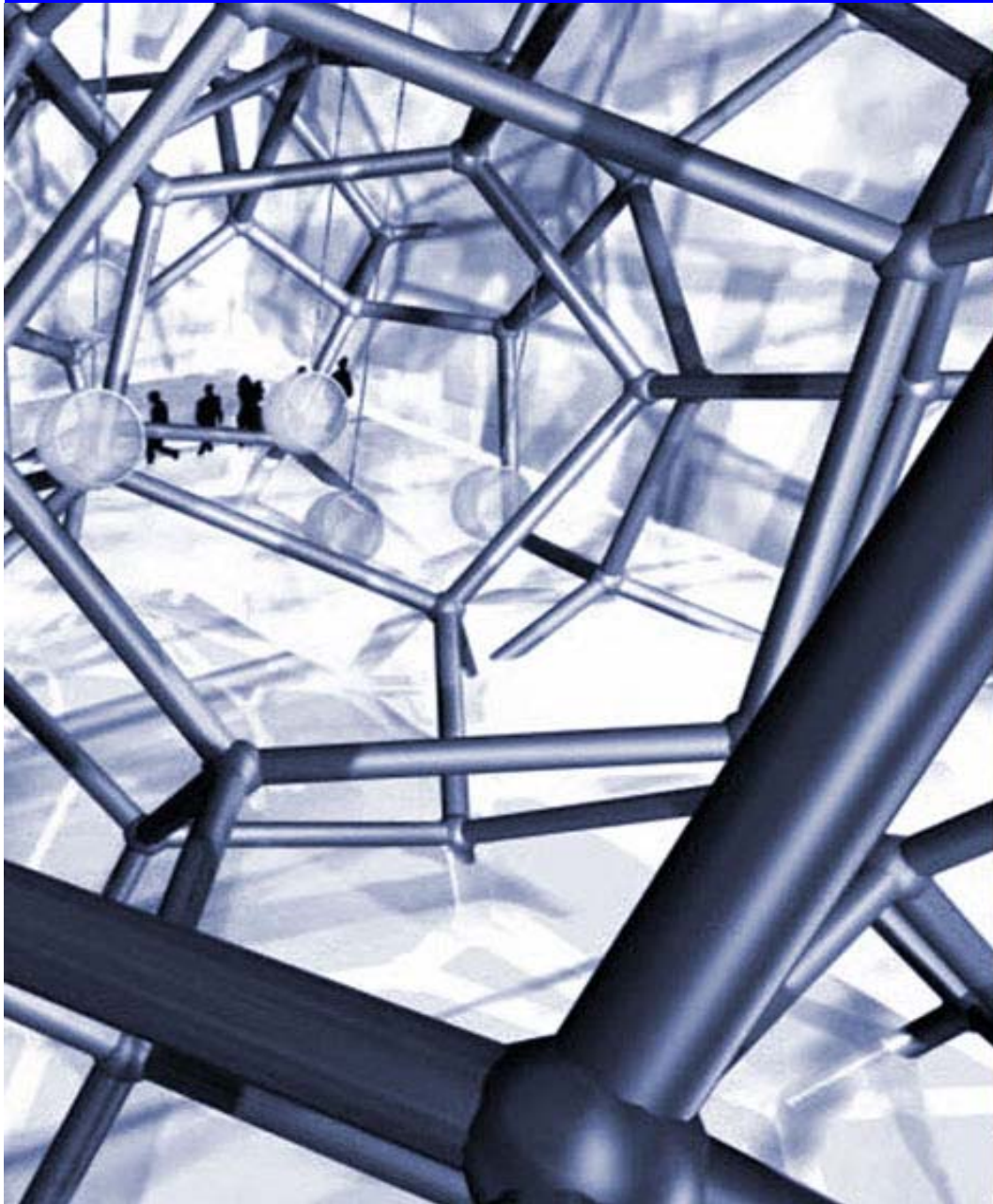
Tristram Carfrae

6500 tonnes of steel

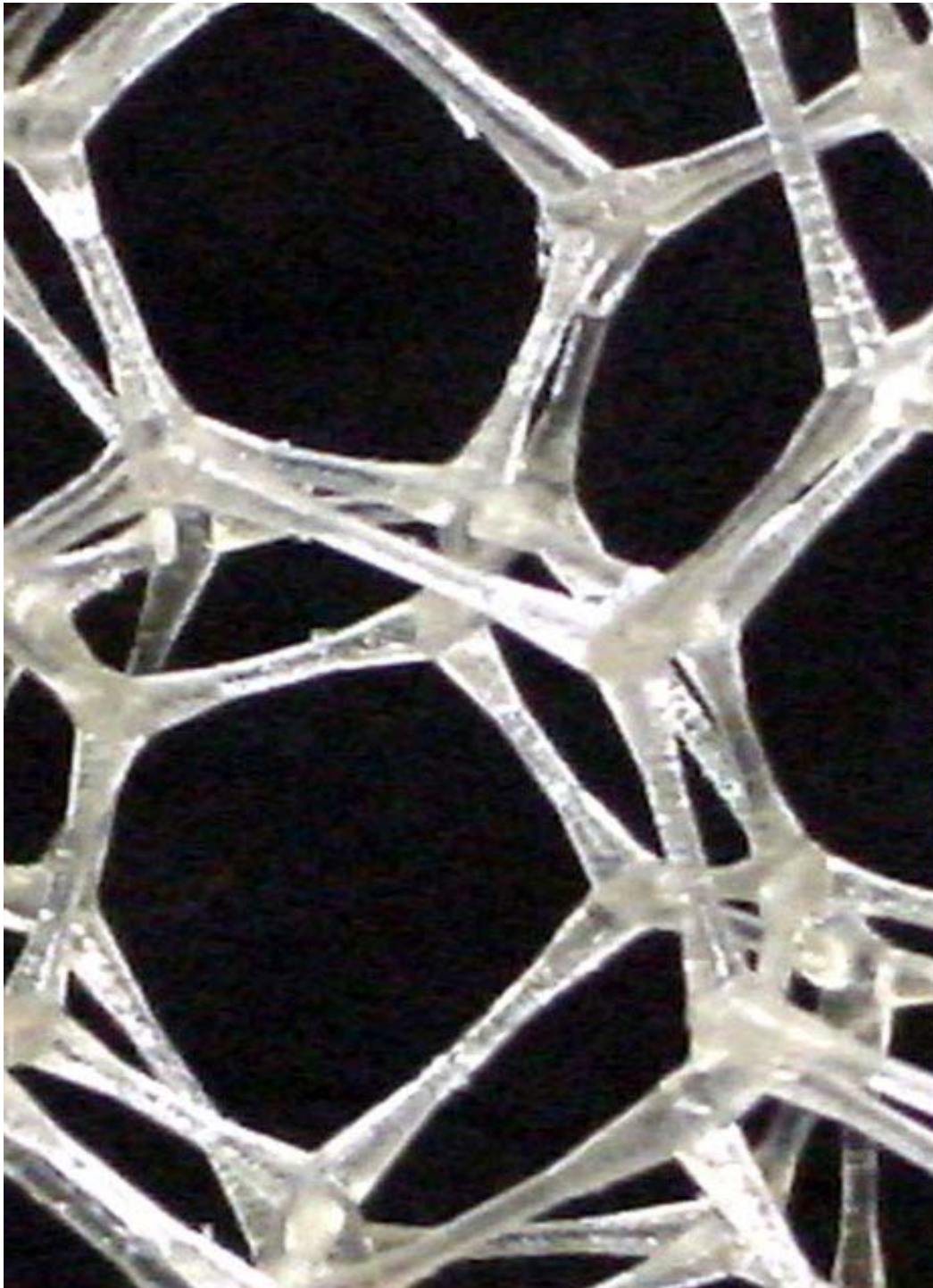
Steel beams would stretch for 90kms

**4,000 bubbles !**







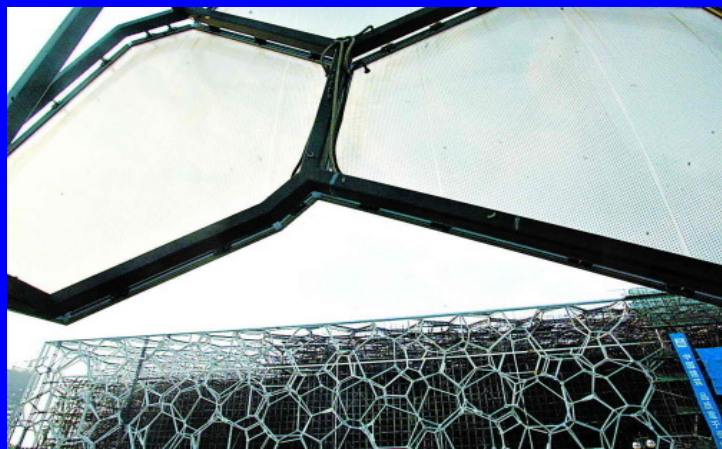
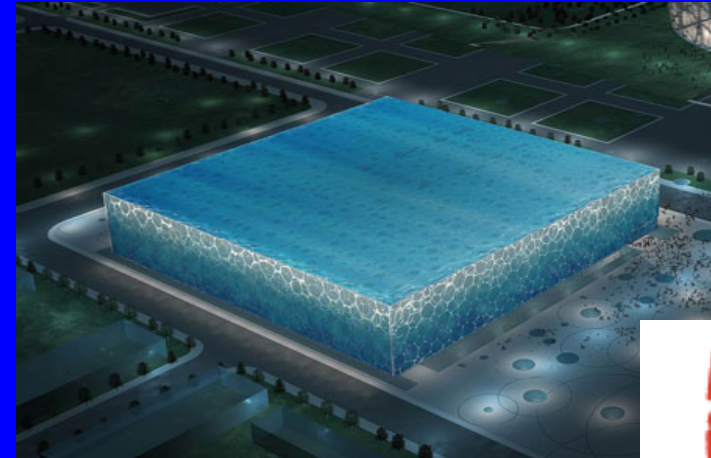








It even looks like it is supposed to do..



<http://www.flickr.com/search/?q=watercube&m=text>

# VIDEO CLIP

April 2007 Beijing Olympic buildings

<http://educatedearth.net/video.php?id=3152>

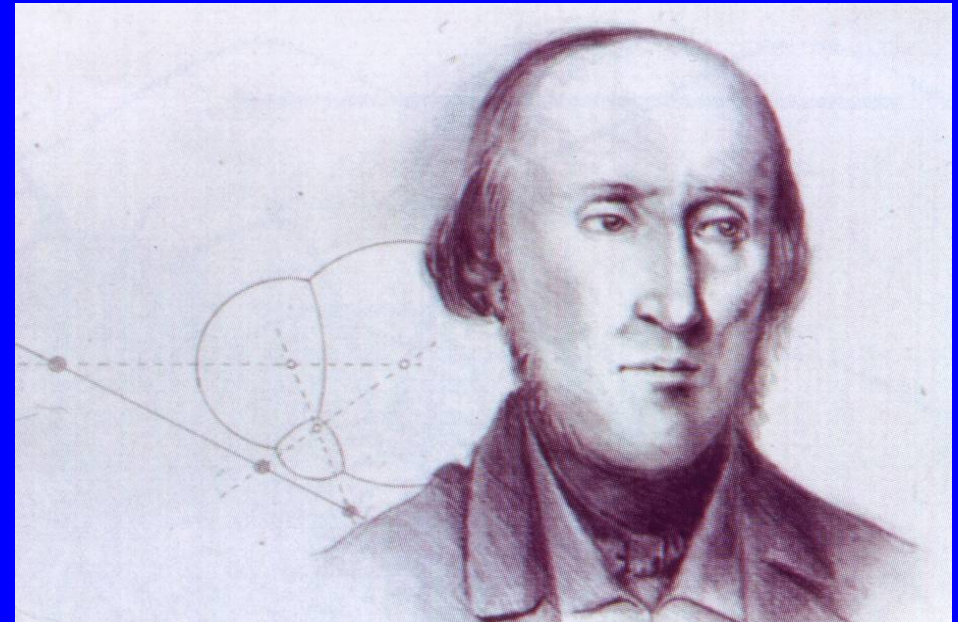
# Joseph Plateau

Blind experimentalist

Classic text 1873

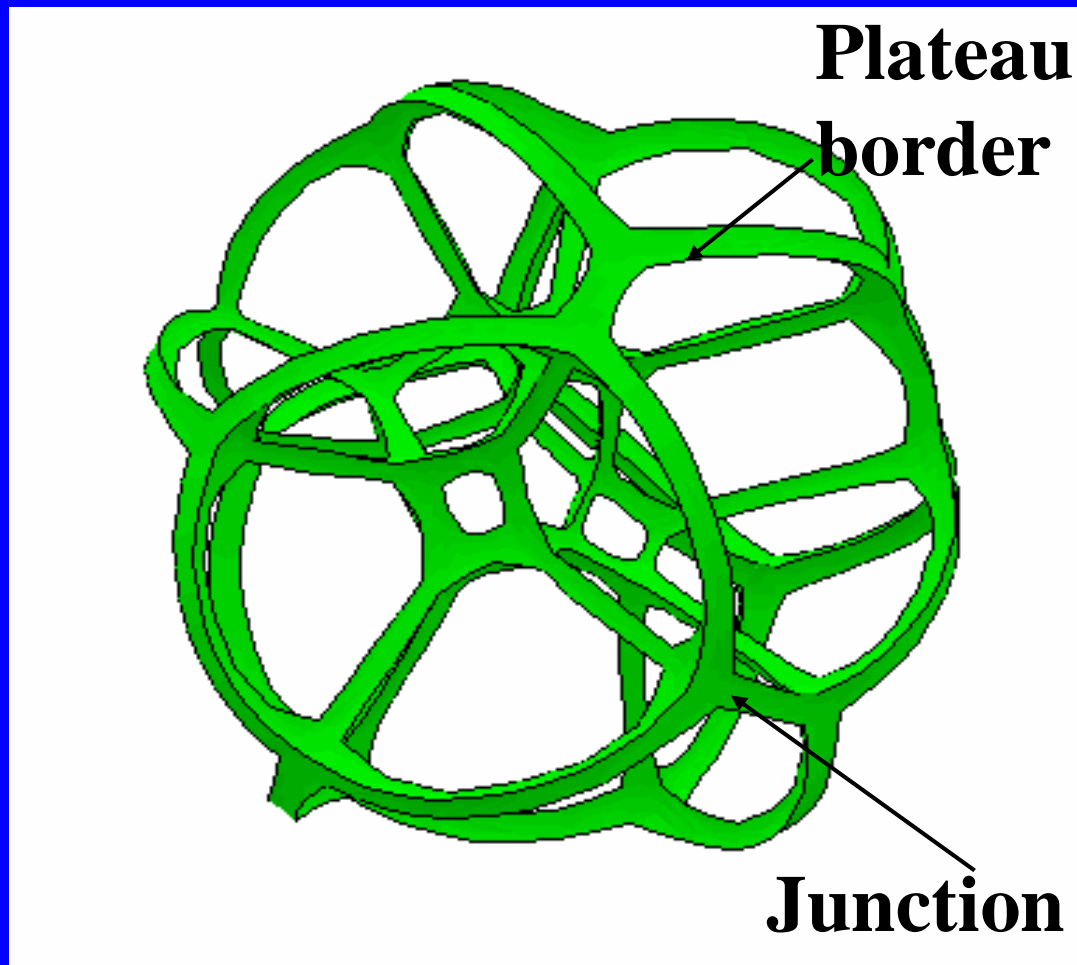
Rules of equilibrium

Wire frame demos

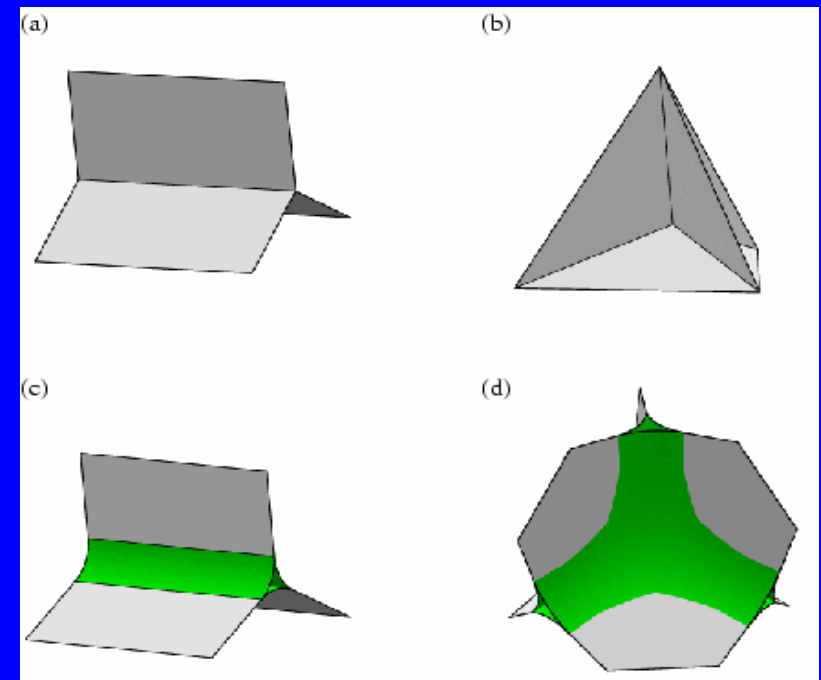




# Elements of the foam structure



## Dry Foam

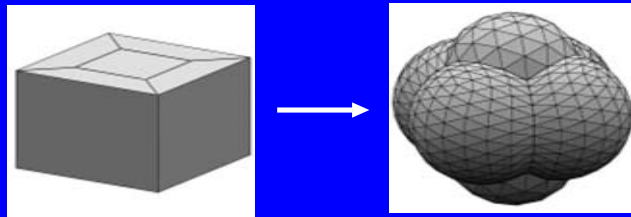


## Wet Foam

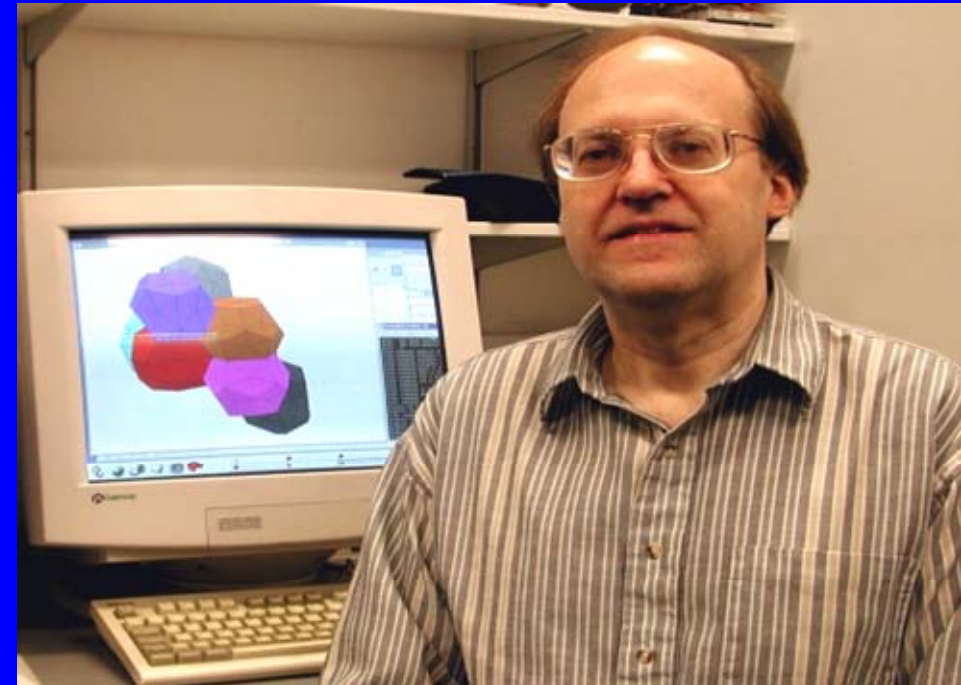
# Ken Brakke

Mathematician

Surface Evolver

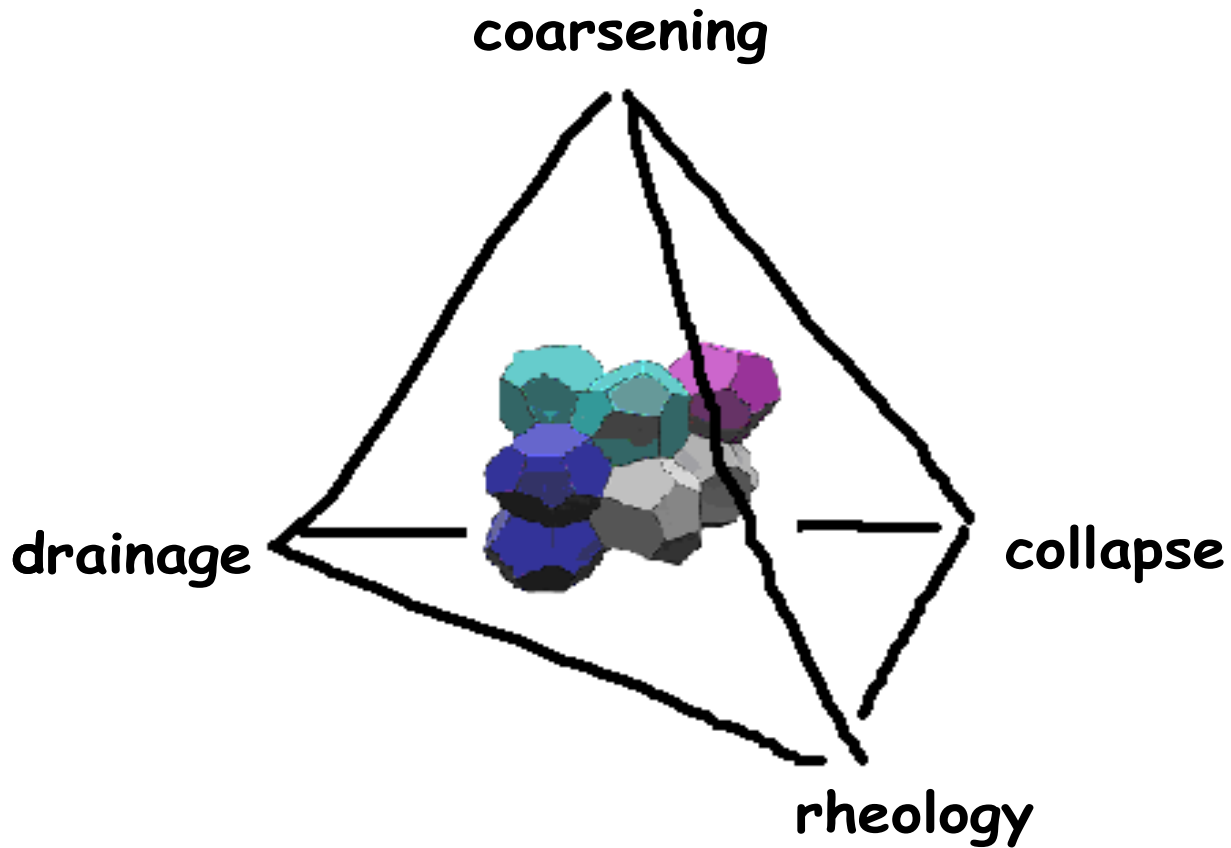


<http://www.susqu.edu/facstaff/b/brakke/>



John Sullivan, Andy Kraynik

*The evolver is a spectacular example of the effects of a gift to science which advances a whole field.* Alan L. Mackay





# Cyril Stanley Smith

Metallurgist

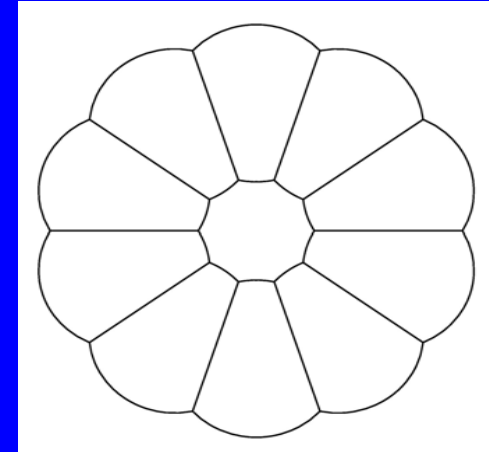
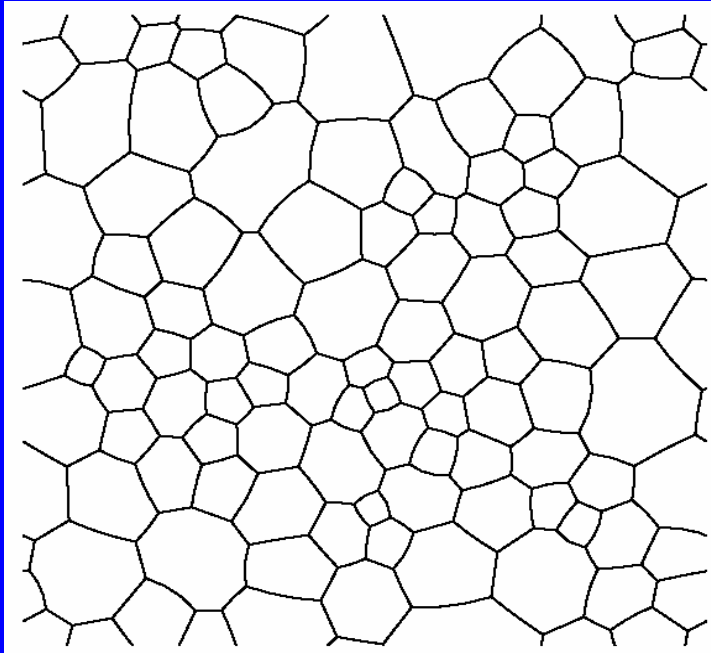
2D soap froth

Model for grain growth

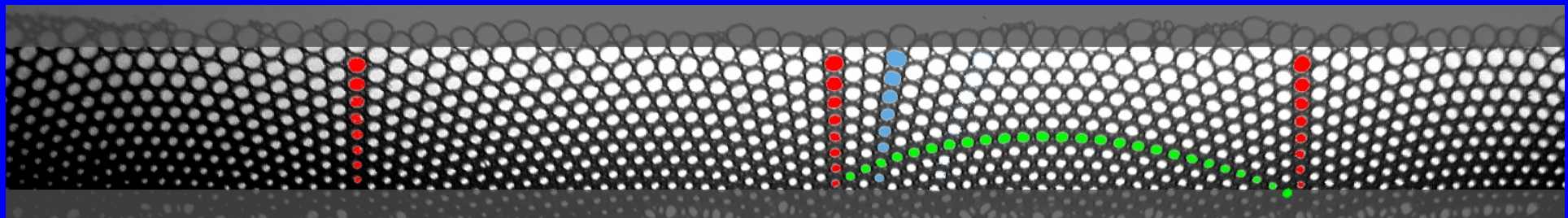
Inspiring influence!



# Two – dimensional foams



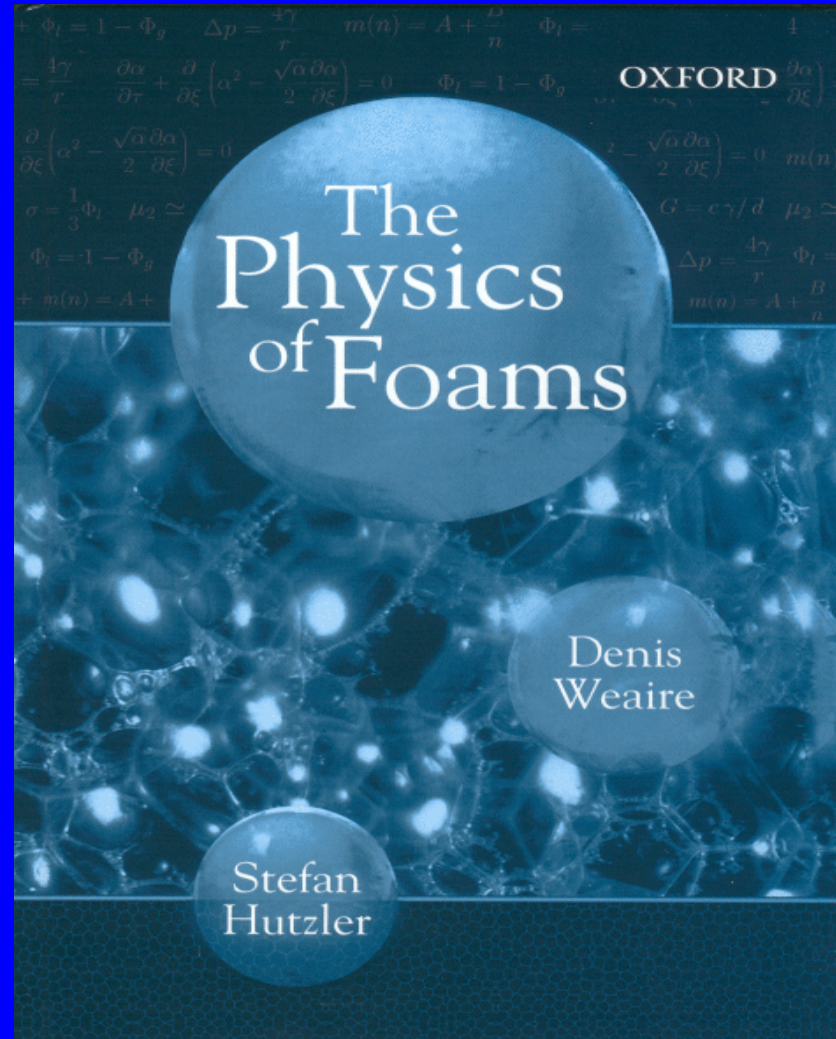
Gravity's rainbow



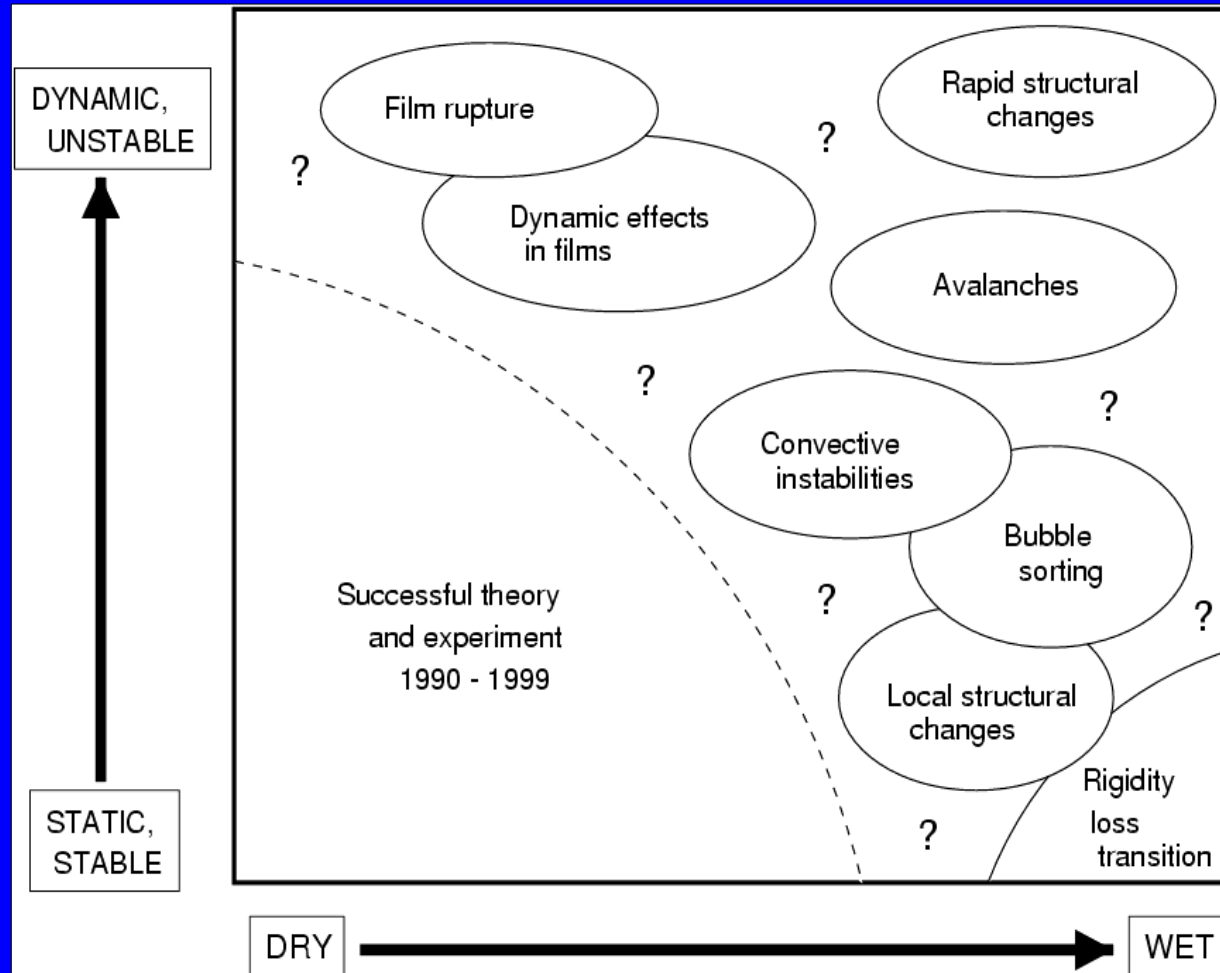
# From statics to dynamics...

Statics and  
quasistatics:  
structure, elasticity,  
coarsening

Dynamics: drainage  
and rheology



# New frontiers in foams





# The Challenge to Theory and Simulation

?

Local dynamics of bulk liquid, surfaces and gases

Chemistry, surface chemistry

?

Dynamics of structured elements: films, Plateau borders and junctions

Physics, local rheology

?

Continuum description of bulk properties: drainage, **rheology** (coarsening, collapse)

*RHEOLOGY*





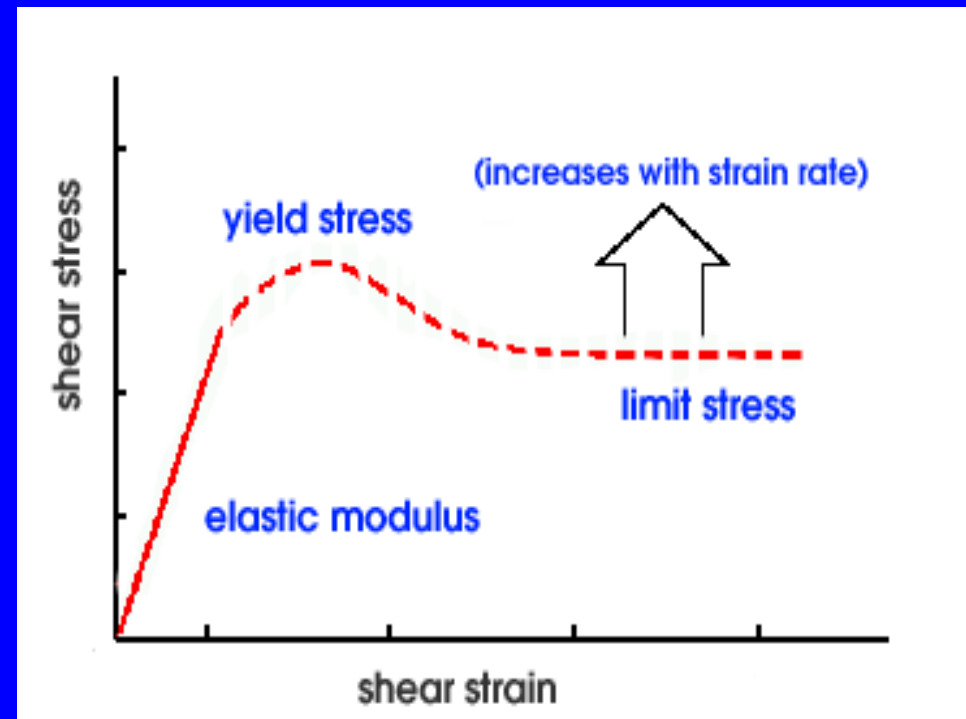
# Rheology with yield stress

Steady shear

Yield stress=limit stress?

Stress = limit stress + X

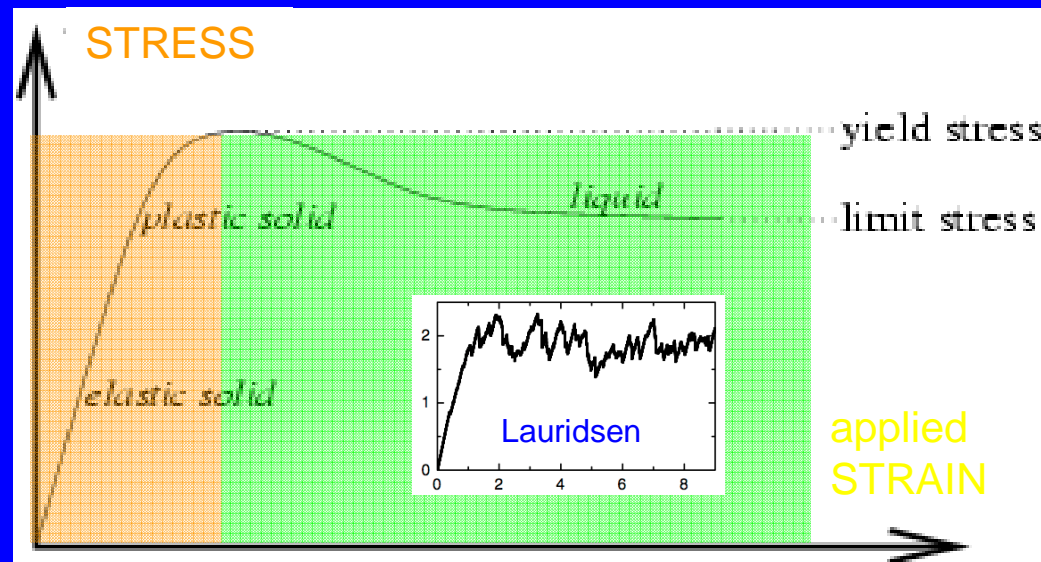
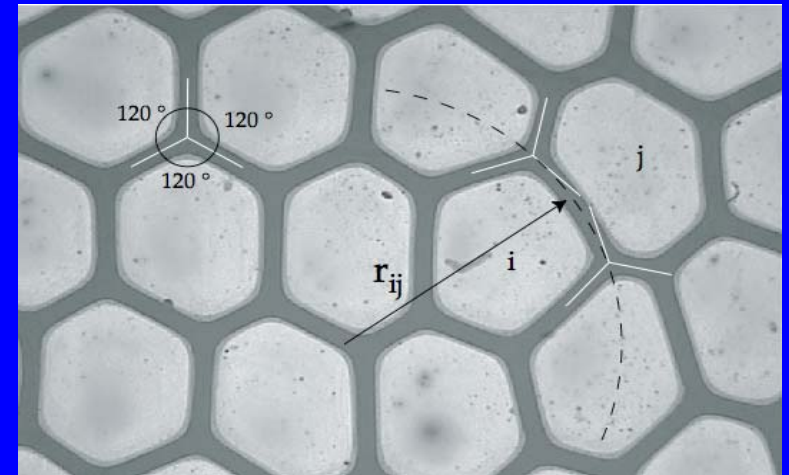
$X \sim (\text{strain rate})^n, n = ?$



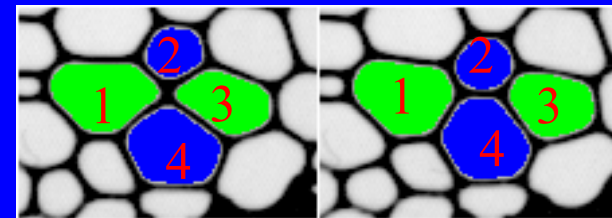


# (quasi) 2D foams as a model system for foam rheology

- simpler and easier than 3D foams!
- same generic mechanical properties
  - elastic at small strain
  - flow at large strain (hysteretic!)
  - funneous?



T1 events: **plasticity**

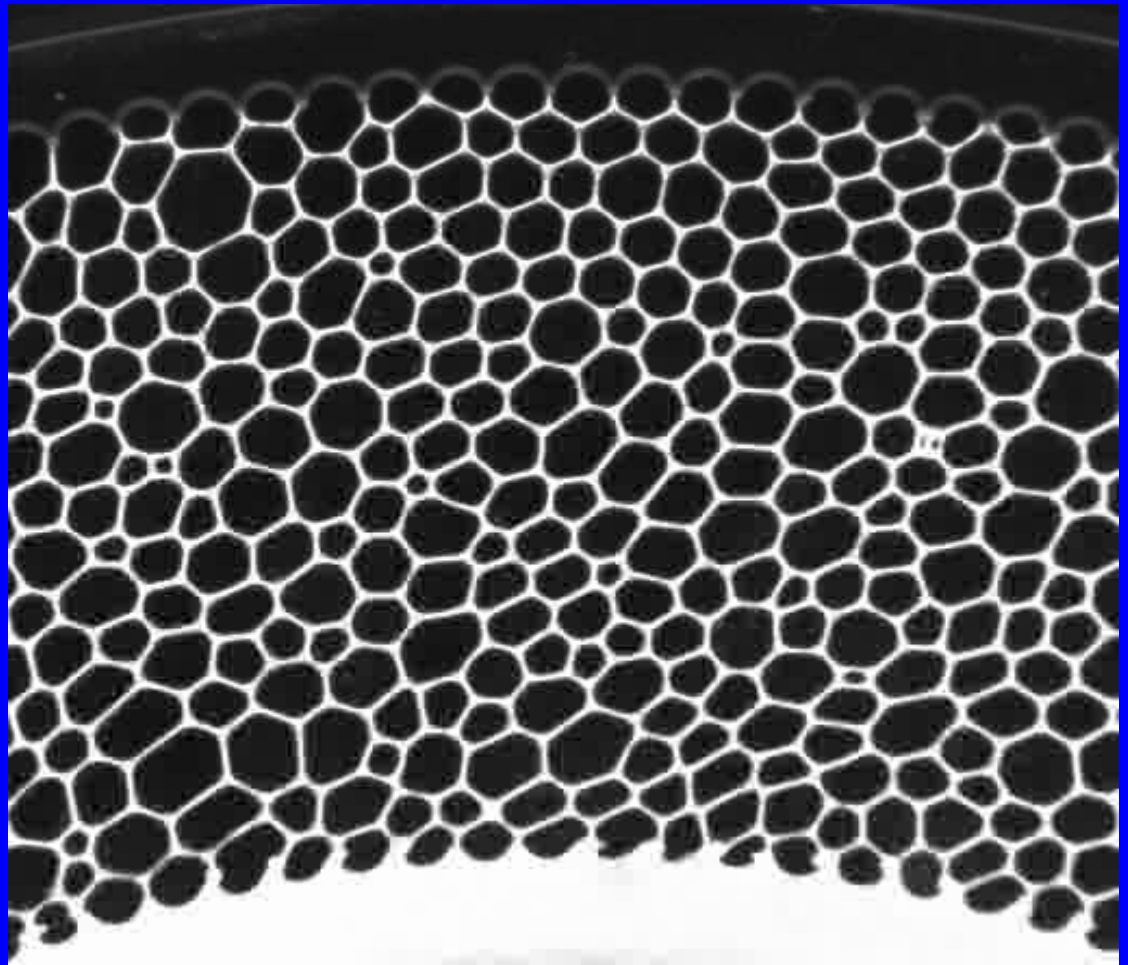


# The Experiment of Debregeas

Between glass plates

Localisation

G.Debregeas *et al*,  
Physical Review Letters  
8717,8305 (2001)



# A 2d effect: wall drag

$$F \sim v^n$$

Mobile Surface

$$n = 2/3$$

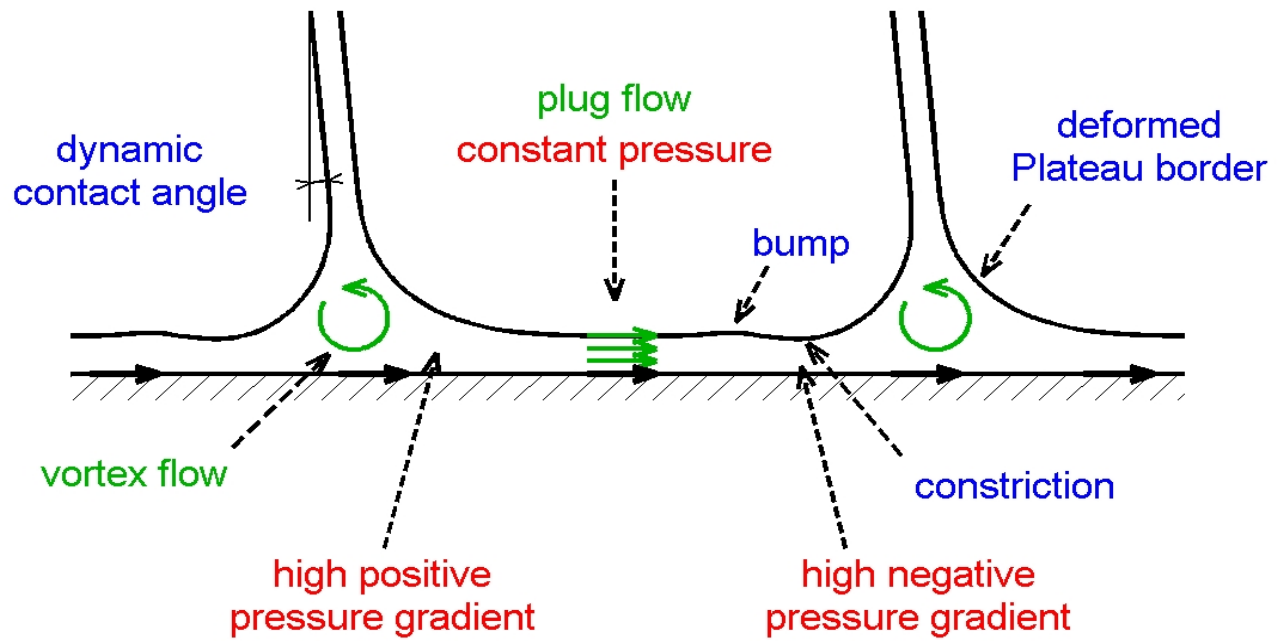
Bretherton

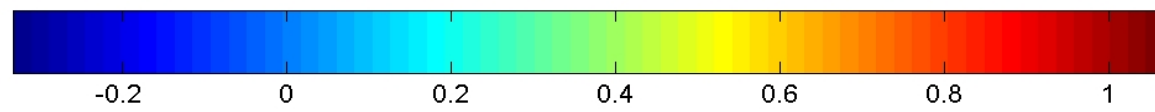
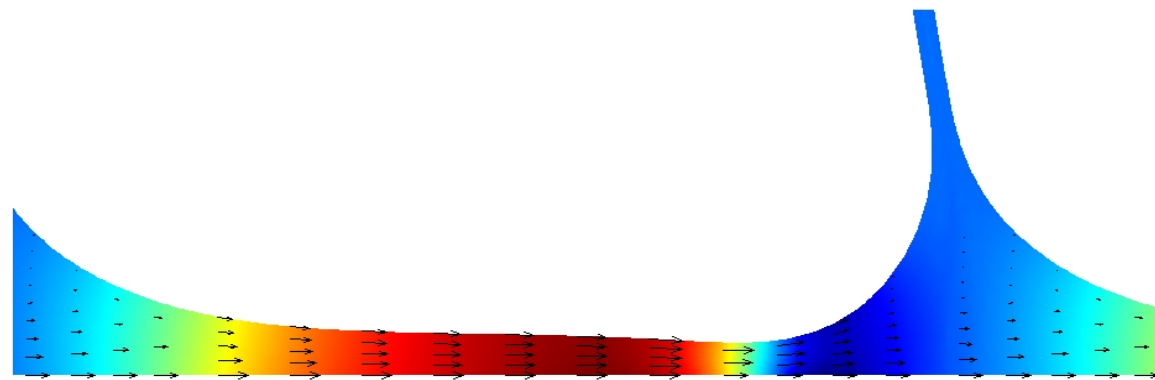
Immobile Surface

$$n = 1/2$$

Denkov *et al.*







# A continuum model?

Detailed quasistatic simulations have shown localisation (shear banding) but with some anomalous features.

Why not attempt a continuum description?



# Four key ingredients:

Bingham-like constitutive equation:

$$\sigma = \sigma_0 f(\gamma/\gamma_0) + \eta \dot{\gamma}$$

Elastic

Plastic

viscous

$$f(x) = \tanh(x)$$

(linear) Viscous drag:

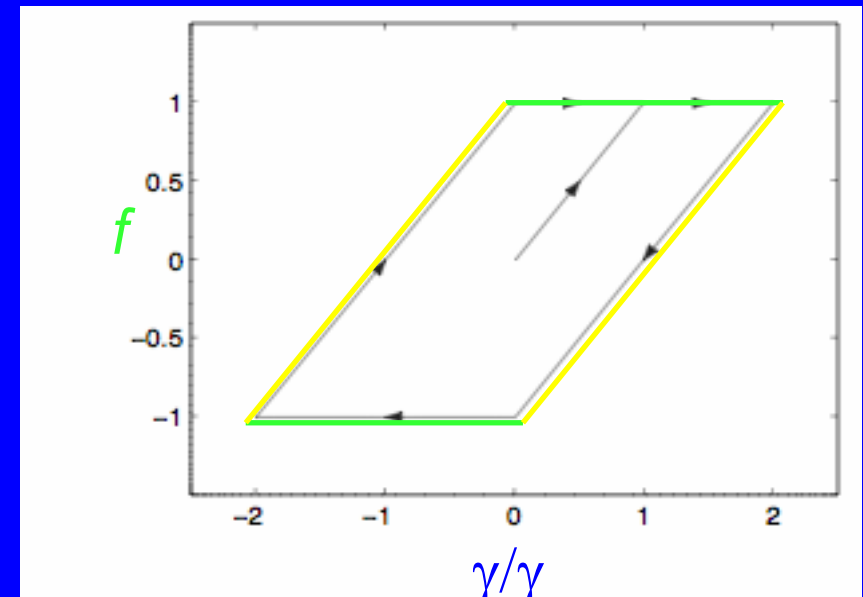
$$F = \beta v$$

$\sigma_0$  yield stress

$\eta$  Bingham viscosity

$\beta$  viscous drag coefficient

hysteretic cycle



Scaling parameters of the fluid:

characteristic length:

$$L_0 = \sqrt{\frac{\eta}{\beta}}$$

characteristic velocity:

$$V_0 = \frac{\sigma_Y}{\sqrt{\eta\beta}}$$

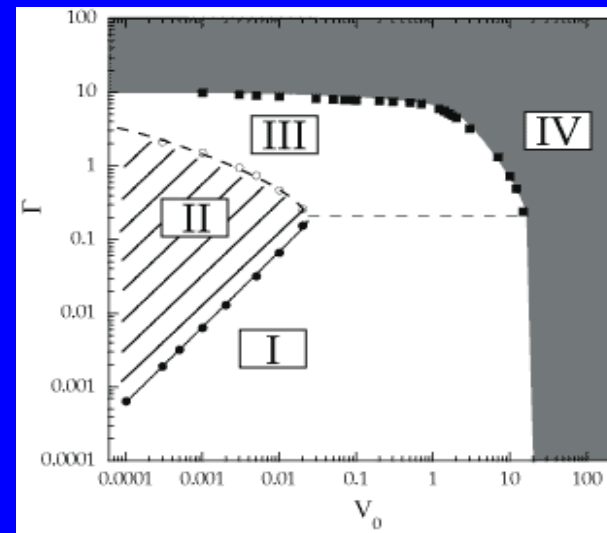
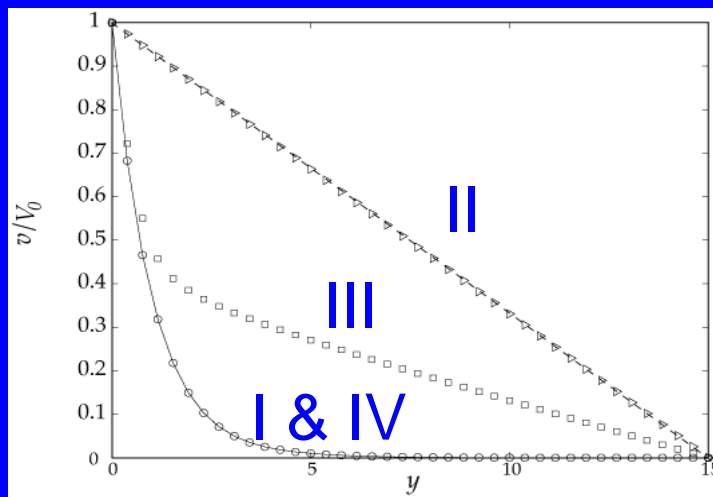
# Hence a simple continuum model for quasi 2D foams

- simple constitutive equation

$$\sigma = \sigma_y \tanh \gamma + \eta \dot{\gamma}$$

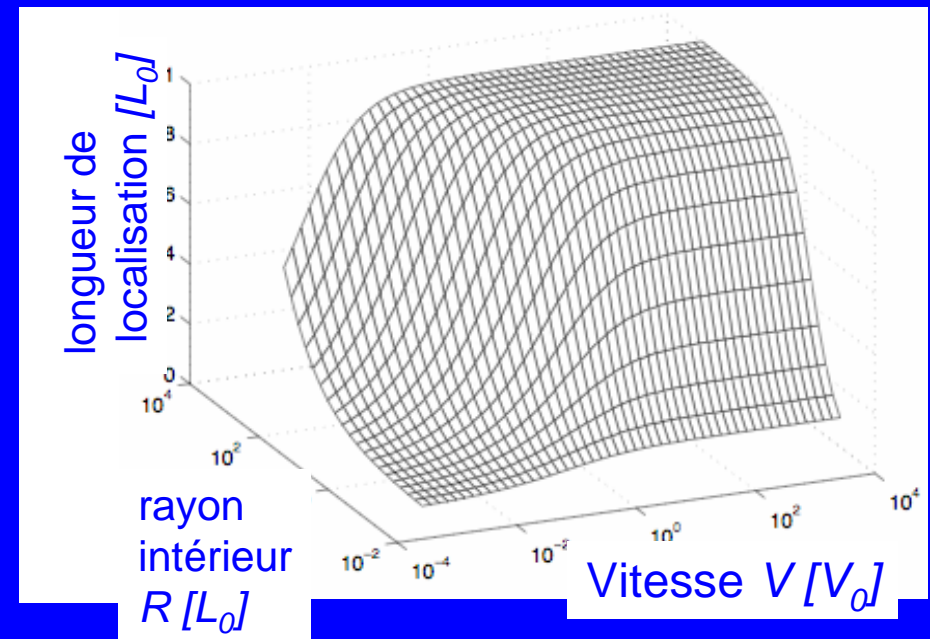
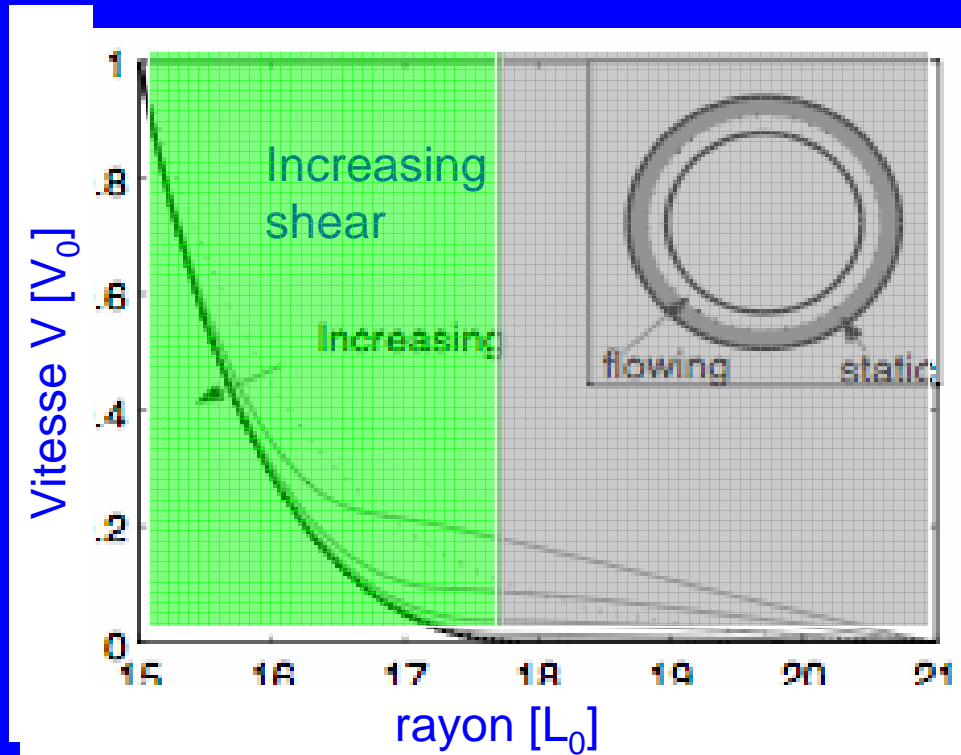
- adding viscous drag for quasi 2D foams:

$$\text{div} \sigma = \beta v$$



# Couette geometry: inner rotating boundary

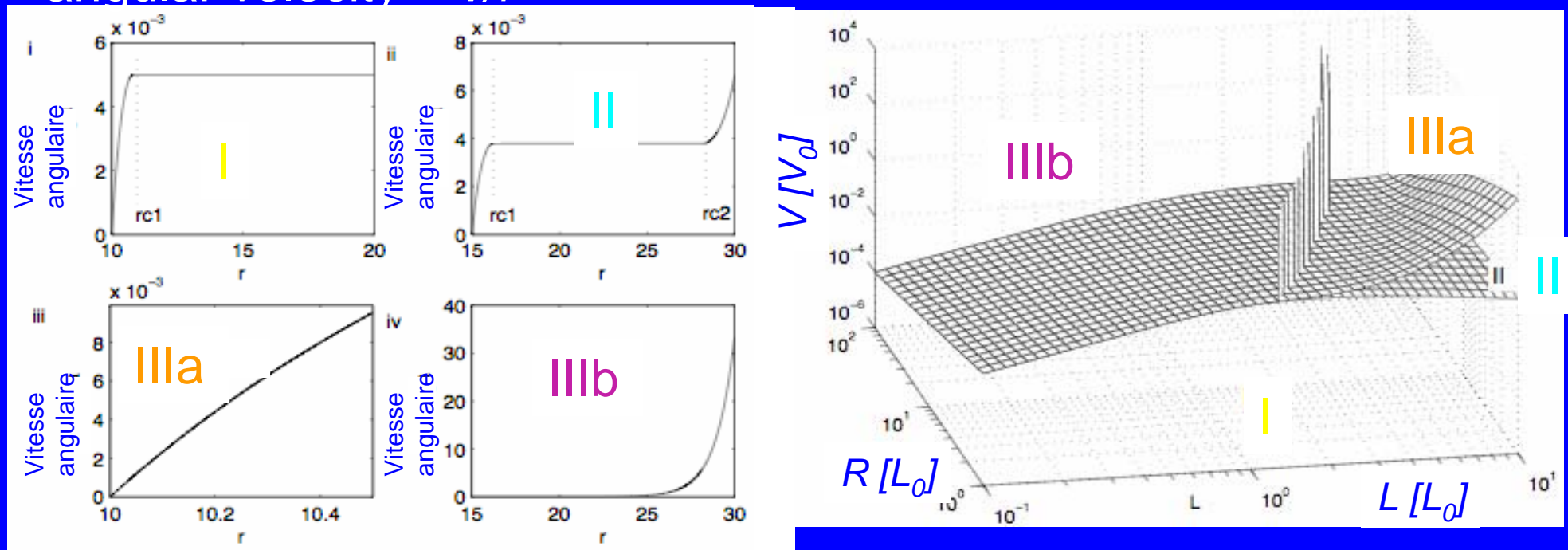
Localisation length varies





# Outer moving boundary

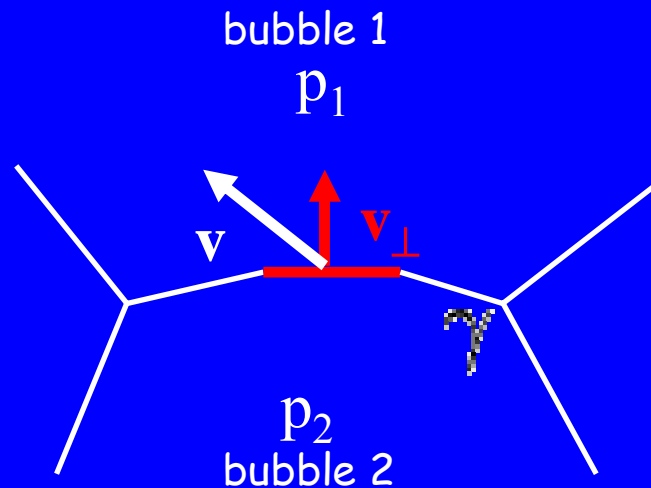
angular velocity =  $v/r$



**I** et **IIIa** experimentally observed (Lauridsen prl 2004 & 2002)

**IIIa** et **IIIb** correspond to the straight edge case

# Dynamics with drag?



$P_i$  - pressure in bubble I

$\gamma$  - surface tension

$c$  - curvature

$\lambda$  - drag coefficient

$v_{\perp}$  - normal velocity

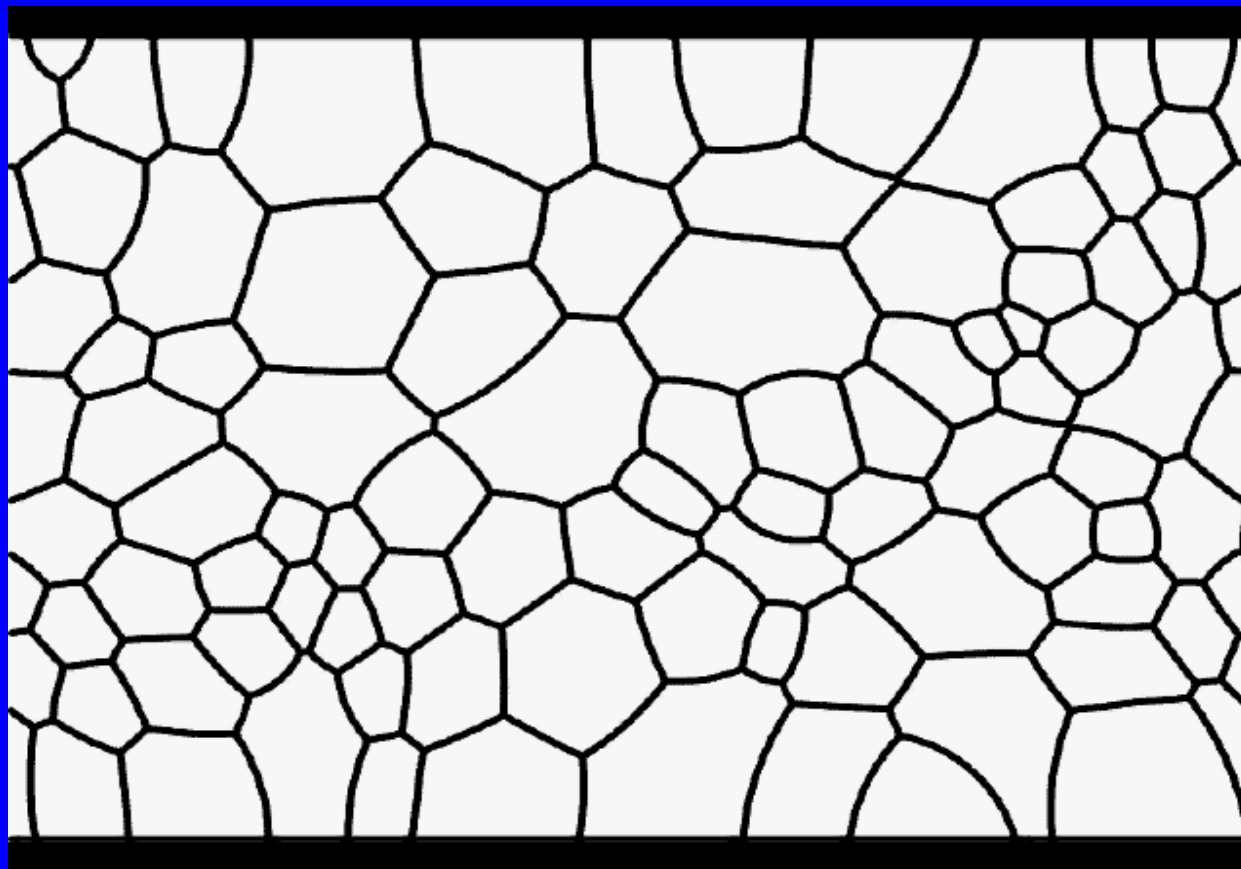
Laplace law:

$$P_1 - P_2 = \gamma c$$

Viscous froth:

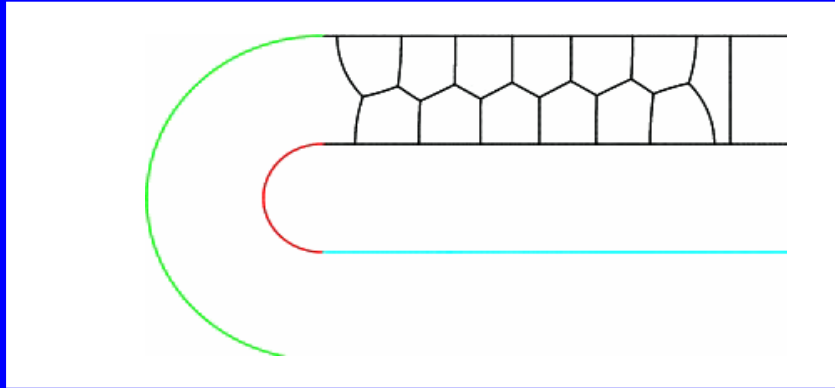
$$P_1 - P_2 = \gamma c - \lambda v_{\perp}$$

## Viscous Froth 2

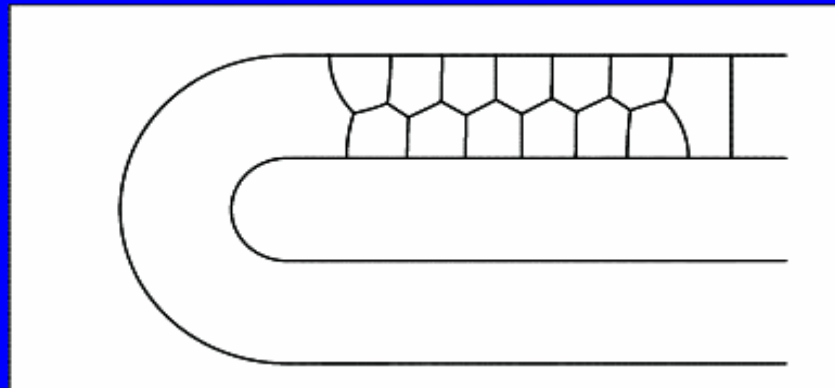




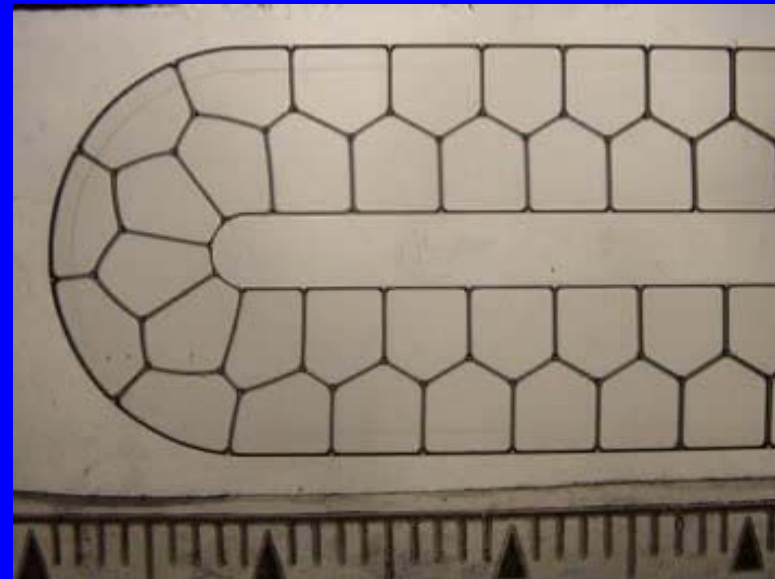
# Geometric effects in channels



quasi – static --> no T1

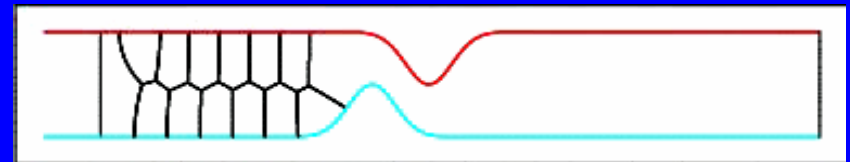
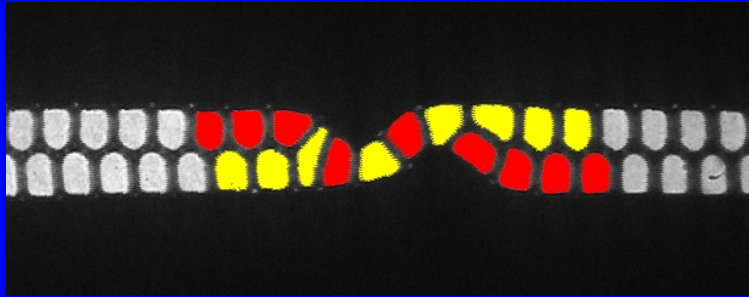


viscous froth --> T1

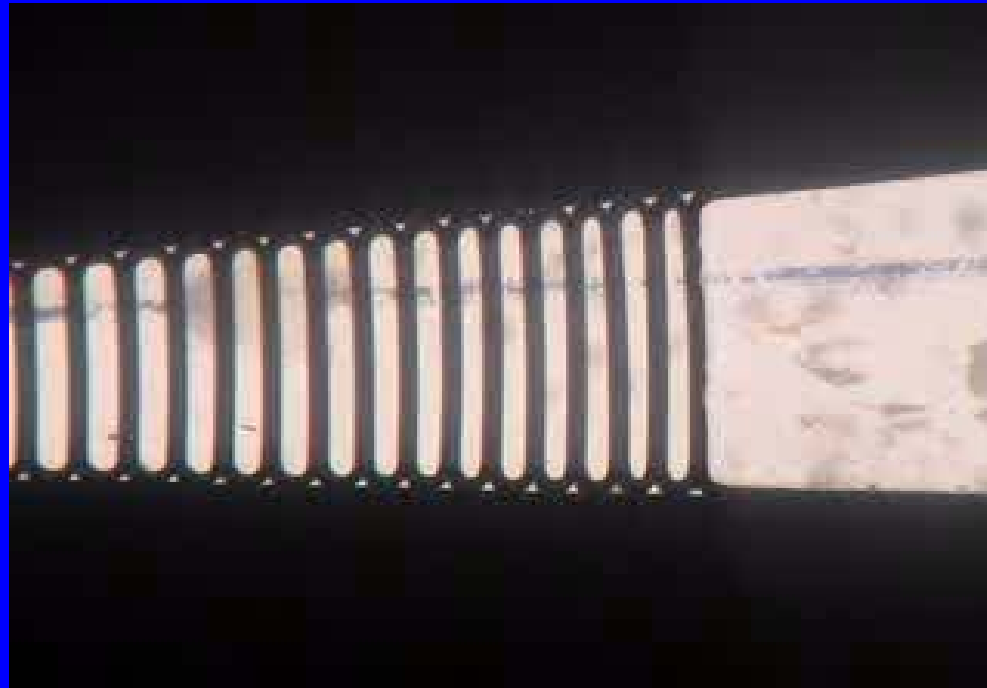


Experiment  
increase velocity

# The Flipper



# The Zipper



# The Y-junction

